 

**Refutation of Gödel Program with ZFC and Hilbert's first continuum problem without ZFC**

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ , | ; - Not Or; & And, ∧, ∩, ⊓, ·, ∘ , ⊗ ; \ Not And, ↑ ;
> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ←, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠, ⊕ ;
% possibility, for one or some, ∃, ∃!, ◊, M ; # necessity, for every or all, ∀, □, L ;
(z=z) T as tautology, ⊤, ordinal 3 ; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero ;
(%z>#z) N as non-contingency, Δ, ordinal 1 ; (%z<#z) C as contingency, ∇, ordinal 2 ;
~( y < x) ( x ≤ y), ( x ⊆ y), ( x ⊑ y); ~( x < y) ( x ≥ y); (A=B) (A~B).
Notes: for clarity, we usually distribute quantifiers onto each designated variable; and for ordinal arithmetic, the result is implied.

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Abstract. Gödel proved in the 1930s in his famous Incompleteness Theorems that not all statements in mathematics can be proven or disproven from the accepted ZFC axioms. A few years later he showed the celebrated result that Cantor's Continuum Hypothesis is consistent. Afterwards, Gödel raised the question whether, despite the fact that there is no reasonable axiomatic framework for all mathematical statements, natural statements, such as Cantor's Continuum Hypothesis, can be decided via extending ZFC by large cardinal axioms. While this question has been answered negatively, the problem of finding good axioms that decide natural mathematical statements remains open. There is a compelling candidate for an axiom that could solve Gödel's problem: $\mathsf{V} = \mathsf{Ultimate} - \mathsf{L}$. In addition, due to recent results the $\mathsf{Sealing}$ scenario has gained a lot of attention. We describe these candidates as well as their impact and relationship.

**Example: Cantor's Continuum Problem.** Arguably the most famous statement that is known to be independent from ZFC is Cantor's Continuum Problem. It was formulated by Cantor in 1878 and appeared as the first item on Hilbert's list of problems announced at the International Congress of Mathematicians in Paris in 1900. Informally, it can be phrased as the question how many real numbers there are. Or, a bit more formally, as the following question: Is there a set $A$ of size strictly between the size of the set of natural numbers $|\mathbb{N}|$ and the size of the set of real numbers $|\mathbb{R}|$? I.e., is there a set $A$ such that

$$|\mathbb{N}| < |A| < |\mathbb{R}|?$$

[continued on next page]

We define the sets as absolute values of the usual way.

LET p, q, r: | N |, | A |, | R | .

Ordinal one is defined as (%s>#s), and zero is defined as (s@s).
One as proof (s=s) can also be described as zero for not contradiction ~(s=s) or (s@s).

The numbers for p, q, r are restricted where

Natural numbers are integers greater than zero, that is not less than one, and the other sets are numbers greater than zero, that is not equal to zero. This complies with zero as a marker. (1.1)

(~(p<(%s>#s))&(q>(s@s)))&(r>(s@s)) ; TN**FF** **FFFF** TN**FF** **FFFF** (1.2)

(~(p<(s=s))&(q>(s@s)))&(r>(s@s)) ; TT**FF** **FFFF** TT**FF** **FFFF** (1.3)

All numbers are also not equivalent to each other for unique sets. (2.1)

~(((p=q)+(p=r))+(q=r)) = (s=s) ; **FFFF** **FFFF** **FFFF** **FFFF** (2.2)

The antecedent is formed with the range description of Eq. 1.2 or of 1.3 to imply the non-equivalence description of 2.2 as: (3.1, 3.2)

((~(p<(%s>#s))&(q>(s@s)))&(r>(s@s))) > ~(((p=q)+(p=r))+(q=r)) ;
**F**CTT **FFFF** **F**CTT **FFFF** (3.3)

((~(p<(s=s))&(q>(s@s)))&(r>(s@s))) > ~(((p=q)+(p=r))+(q=r)) ;
**FF**TT TTTT **FF**TT TTTT (3.4)

The consequent is formed in dividing p, q, r by p to indicate that the natural numbers as p\p imply ordinal one or proof one, also to mean the least value in the relations is an integer identity of one. (4.1, 4.2)

((p\p)>(%s>#s))<((q\p)<(r\p)) ; NTNT N**F**NT NTNT N**F**NT (4.3)

((p\p)>(s=s))<((q\p)<(r\p)) ; TTTT T**F**TT TTTT T**F**TT (4.4)

The arguments for the two renditions of one are 3.3 implies 4.3 or 3.4 implies 4.3 (5.1, 5.2)

(((~(p<(%s>#s))&(q>(s@s)))&(r>(s@s))) > ~(((p=q)+(p=r))+(q=r))) >
(((p\p)>(%s>#s))<((q\p)<(r\p))) ; TTNT N**F**NT TTNT N**F**NT (5.3)

(((~(p<(s=s))&(q>(s@s)))&(r>(s@s))) > ~(((p=q)+(p=r))+(q=r))) >
(((p\p)>(s=s))<((q\p)<(r\p))) ; TTTT T**F**TT TTTT T**F**TT (5.4)

**Remarks:** Eqs. 5.3 and 5.4 are both *not* tautologous to refute Cantor's continuum (1878) as Hilbert's first problem (1900). For one as (s=s) in 5.4, the truth table is closer to tautology, but still no cigar.

This exercise in bivalent logic refutes Gödel's program with ZFC and consistency of the continuum hypothesis without ZFC: incompleteness theorems also fail due to ZFC's ironic contradiction. This resets the foundations of mathematics to Boole's inadvertent *first* discovery of modal logic (1850).